Prediction of Turbulent Separated Boundary Layers

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Theme

AN integral boundary-layer method is extended to calculation of separated turbulent boundary layers by treating the pressure as a dependent variable and prescribing the wall shear variation. The boundary-layer method and a suitable potential flow method are used in an iterative procedure to produce a method for predicting the characteristics of separated flows. The interaction between the boundary layer and the inviscid flow is accounted for by augmenting the physical surface by the boundary-layer displacement thickness. Good comparisons are shown between the theory and data for a separated turbulent boundary layer on the wall of a transonic wind tunnel.

Contents

Accurate prediction of the occurrence of boundary-layer separation is important in a wide variety of aeronautical applications. Many attempts have been made to develop criteria for predicting the occurrence of separation by using analytical and empirical methods. A recent evaluation of these criteria indicates that none of them is generally applicable to a wide range of flow conditions[1] and none of them allow prediction of the boundary-layer characteristics in the separated region.

In this paper, a technique is described for predicting the viscous-inviscid interaction of separated turbulent boundary layers taking account of the elliptical nature of the problem. The technique combines an integral boundary-layer method developed by the authors and a finite-difference potential flow method described in Ref. 2. The boundary-layer characteristics and the inviscid flowfield are calculated by an iteration procedure in which the boundary layer and inviscid flow are calculated alternately by separate calculative programs. In attached flow regions, the pressure calculated from the inviscid flow is prescribed for the boundary-layer calculation, and the displacement thickness of the boundary layer is used to augment the surface coordinate to provide a boundary condition for the inviscid flow. In the separated region, the pressure cannot be prescribed in the boundary-layer calculations. Instead, the wall shear stress is prescribed and the corresponding pressure and displacement thickness are calculated from the boundary-layer method. The iterative process is continued until the pressure distributions produced by the viscous solution and the inviscid solution agree.

The basic boundary-layer calculation method used in this work uses the equations of a steady, incompressible, fully turbulent flow with no normal pressure gradients with the Reynolds stresses represented by an eddy viscosity. The integral method chosen to calculate the boundary layer was described in detail in Ref. 3. Families of integral equations are derived by combining the momentum and continuity equations and then taking weighted integrals of the resulting equation across the boundary layer

$$\int_o^\delta \left\{ UU_x - U_y \int_o^y U_x \, d\eta - U_e(U_e)_x - \nu(\beta U_y)_y \right\} f(y)\, dy = 0 \qquad (1)$$

The functions

$$f = y^n; \quad n = 0, 1 \qquad (2)$$

produce the momentum and moment of momentum integral equations, respectively.

The y dependence of the integral equations is eliminated by substituting an appropriate parametric formulation for the velocity profiles. The function used for the present theory is

$$U = u_\tau[2.5 \ln(1+y^+) + 5.1 - (3.39y^+ + 5.1)e^{-0.37y^+}] + \tfrac{1}{2}u_\beta\{1 - \cos[\pi(y/\delta)]\} \qquad (3)$$

The parameter u_τ is the usual friction velocity, modified to accommodate separated flows

$$u_\tau = (\tau_w/|\tau_w|)(|\tau_w|/\rho)^{1/2} \qquad (4)$$

The eddy viscosity model used in this work is a two-layer model using Clauser's expression with intermittency for the outer layer and an exponential expression based on the law of the wall for the inner layer. For separated flows, the scaling of the outer eddy viscosity is a displacement thickness based on the velocity profile above the $U = 0$ line.

Substitution of Eq. (3) into the two equations produced by Eqs. (1) and (2) produces two ordinary differential equations for the variation of the variables u_τ, u_β, δ, and U_e with x. A third equation produced by evaluating Eq. (3) at $y = \delta$ allows the elimination of u_β from the equations, leaving a set of two equations

$$A_{11}(u_\tau)_x + A_{12}\delta_x + A_{13}(U_e)_x = -\nu u_\tau |u_\tau|/U_e \qquad (5)$$

$$A_{21}(u_\tau)_x + A_{22}\delta_x + A_{23}(U_e)_x = -\frac{\nu}{U_e\delta^2}\int_o^\delta \beta u_y \, dy \qquad (6)$$

The velocity U_e is considered to be a dependent variable. The coefficients A_{ij} are functions of the variables u_τ, δ, and U_e. The equations are integrated numerically from known initial conditions and with one of the dependent variables prescribed.

The usual procedure for solving Eqs. (5) and (6) for attached boundary layers is to prescribe the pressure distribution, or the velocity distribution at the edge of the layer. However, if separation occurs, the pressure distribution cannot be prescribed arbitrarily. If an adverse pressure gradient is prescribed for an attached boundary layer, the value of u_τ can approach zero. When u_τ vanishes, the coefficients A_{11} and A_{21} in Eqs. (5) and (6) also vanish, producing a singularity. The singularity is removed by rearranging the equations so that u_τ can be prescribed and U_e (or p) calculated as a dependent variable. The precise manner in which u_τ must be prescribed is determined using an iterative procedure in which the boundary layer and the inviscid flow are calculated alternately until the pressure distribution calculated for the boundary layer agrees with that calculated for the inviscid flow. The iterative procedure becomes predictive by the introduction of a model for the interaction between the viscous and inviscid flows. In this work, the interaction was accounted for by augmenting the surface by the boundary-layer displacement thickness.

A predictive calculation was performed for the boundary layer on the bump described in Ref. 4, shown in Fig. 1. In calculating

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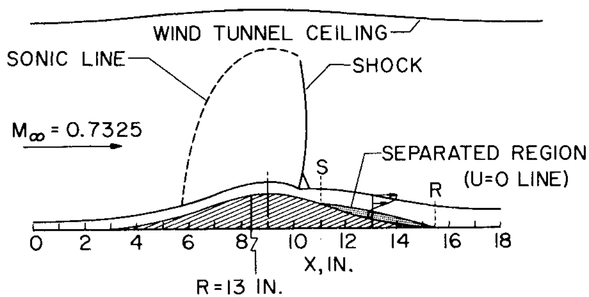

Fig. 1 Transonic flowfield over a bump.[4]

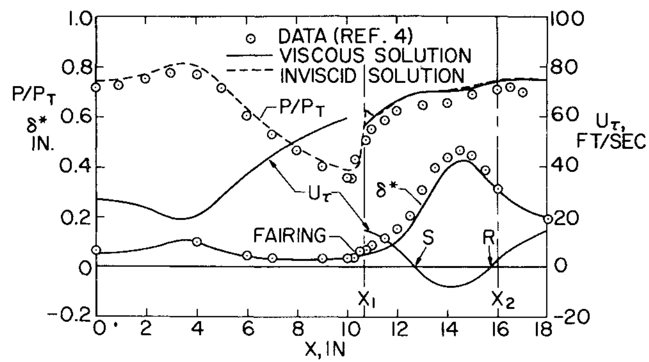

Fig. 2 Converged solution compared with data.

the theoretical flow, the only experimental information used was the velocity profile at the beginning of the test section and an approximate variation of δ^* through the shock wave. The theory of Ref. 2 was used for the inviscid flowfield. That theory is based on transonic small disturbance theory using a line relaxation method to solve the finite-difference equations. For the boundary-layer calculation, the beginning of the region of prescribed u_τ was chosen as a point immediately after the shock wave.

The iteration begins with the calculation of the inviscid flow on the surface with no boundary layer. Next the boundary layer on the surface is calculated. The displacement thickness is then added to the surface and a new inviscid flowfield is calculated for the augmented surface. This process is repeated until convergence is achieved. For attached boundary layers, convergence is rapid, based on decreasing difference between successive solutions. For separated boundary layers, the δ^* distributions must be calculated in two parts. The pressure is prescribed in regions of attached flow while u_τ is prescribed in regions of separated flow. Convergence is achieved when the pressure distributions calculated by the inviscid program and the boundary-layer program with u_τ prescribed agree within a desired tolerance.

In Fig. 2 are shown the results for a "converged" solution. The viscous and inviscid solutions agree within 1%. For comparison, the experimental δ^* and pressure distribution are also shown in Fig. 1. The predicted pressure is seen to be slightly higher than the experimental data due to discrepancies between the wind-tunnel ceiling contour and that of a free streamline as required by the inviscid theory. The predicted δ^* is in excellent agreement with the data upstream of the shock wave. The predicted δ^* for the converged solution downstream of the shock is slightly lower than the δ^* data probably due to slight uncertainty in specifying the boundary-layer conditions downstream of the shock.

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